Euler equation proof

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Abstract

Generated by the Physics Derivation Graph.

Eq. 1 is an initial equation.

$$y = \cos(x) + i\sin(x) \tag{1}$$

Differentiate Eq. 1 with respect to x; yields Eq. 2.

$$\frac{d}{dx}y = -\sin(x) + i\cos(x) \tag{2}$$

Factor i from the RHS of Eq. 2; yields Eq. 3.

$$\frac{d}{dx}y = (i\sin(x) + \cos(x))i \tag{3}$$

Substitute RHS of Eq. 1 into Eq. 3; yields Eq. 4.

$$\frac{d}{dx}y = yi \tag{4}$$

Multiply both sides of Eq. 4 by dx; yields Eq. 5.

$$dy = yidx \tag{5}$$

Divide both sides of Eq. 5 by y; yields Eq. 6.

$$\frac{dy}{y} = idx \tag{6}$$

Indefinite integral of RHS of Eq. 6 over y; yields Eq. 7.

$$\log(y) = idx \tag{7}$$

Indefinite integral of RHS of Eq. 7 over x; yields Eq. 8.

$$\log(y) = ix \tag{8}$$

Swap LHS of Eq. 8 with RHS; yields Eq. 9.

$$ix = \log(y) \tag{9}$$

Make Eq. 9 the power of e; yields Eq. 10.

$$\exp(ix) = y \tag{10}$$

Substitute RHS of Eq. 10 into Eq. 1; yields Eq. 11.

$$\exp(ix) = \cos(x) + i\sin(x) \tag{11}$$

Eq. 11 is one of the final equations.

References