Maxwell equations to electric field wave equation

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March 27, 2024

Abstract

Generated by the Physics Derivation Graph.

Eq. 1 is an initial equation.

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \tag{1}$$

Eq. 2 is an initial equation.

$$\vec{\nabla} \times \vec{H} = \epsilon_0 \frac{\partial}{\partial t} \vec{E} \tag{2}$$

Eq. 3 is an initial equation.

$$\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0 \tag{3}$$

Partially differentiate Eq. 2 with respect to t; yields Eq. 4.

$$\vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} = \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{4}$$

Apply curl to both sides of Eq. 1; yields Eq. 5.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \vec{\nabla} \times \frac{\partial \vec{H}}{\partial t} \tag{5}$$

Substitute LHS of Eq. 5 into Eq. 4; yields Eq. 6.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{6}$$

Eq. 7 is an assumption.

$$\rho = 0 \tag{7}$$

Substitute LHS of Eq. 3 into Eq. 7; yields Eq. 8.

$$\vec{\nabla} \cdot \vec{E} = 0 \tag{8}$$

Eq. 9 is an identity.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \tag{9}$$

Substitute LHS of Eq. 9 into Eq. 8; yields Eq. 10.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\nabla^2 \vec{E} \tag{10}$$

Substitute LHS of Eq. 10 into Eq. 6; yields Eq. 11.

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{11}$$

Eq. 11 is one of the final equations.

References