# derivation of Schrodinger Equation 

none

March 31, 2024


#### Abstract

Generated by the Physics Derivation Graph.


Eq. 1 is an initial equation.

$$
\begin{equation*}
k=\frac{2 \pi}{\lambda} \tag{1}
\end{equation*}
$$

Eq. 2 is an initial equation.

$$
\begin{equation*}
\omega=2 \pi f \tag{2}
\end{equation*}
$$

Eq. 3 is an initial equation.

$$
\begin{equation*}
\hbar=h /(2 \pi) \tag{3}
\end{equation*}
$$

Eq. 4 is an initial equation.

$$
\begin{equation*}
p=h / \lambda \tag{4}
\end{equation*}
$$

Eq. 5 is an initial equation.

$$
\begin{equation*}
E=h f \tag{5}
\end{equation*}
$$

Divide both sides of Eq. 2 by $2 \pi$; yields Eq. 6 .

$$
\begin{equation*}
\frac{\omega}{2 \pi}=f \tag{6}
\end{equation*}
$$

Substitute RHS of Eq. 6 into Eq. 5; yields Eq. 7.

$$
\begin{equation*}
E=\frac{h \omega}{2 \pi} \tag{7}
\end{equation*}
$$

Substitute RHS of Eq. 3 into Eq. 7; yields Eq. 8.

$$
\begin{equation*}
E=\omega \hbar \tag{8}
\end{equation*}
$$

Divide both sides of Eq. 8 by $\hbar$; yields Eq. 9 .

$$
\begin{equation*}
\frac{E}{\hbar}=\omega \tag{9}
\end{equation*}
$$

Divide both sides of Eq. 1 by $2 \pi$; yields Eq. 10.

$$
\begin{equation*}
\frac{k}{2 \pi}=\lambda \tag{10}
\end{equation*}
$$

Substitute RHS of Eq. 4 into Eq. 10; yields Eq. 11.

$$
\begin{equation*}
p=\frac{h k}{2 \pi} \tag{11}
\end{equation*}
$$

Substitute RHS of Eq. 11 into Eq. 3; yields Eq. 12.

$$
\begin{equation*}
p=\hbar k \tag{12}
\end{equation*}
$$

Divide both sides of Eq. 12 by $\hbar$; yields Eq. 13.

$$
\begin{equation*}
\frac{p}{\hbar}=k \tag{13}
\end{equation*}
$$

Replace scalar variables in Eq. 13 with equivalent vector variables; yields Eq. 14.

$$
\begin{equation*}
\frac{\vec{p}}{\hbar}=\vec{k} \tag{14}
\end{equation*}
$$

Eq. 15 is an initial equation.

$$
\begin{equation*}
\psi(\vec{r}, t)=\psi_{0} \exp (i(\vec{k} \cdot \vec{r}-\omega t)) \tag{15}
\end{equation*}
$$

Substitute RHS of Eq. 15 into Eq. 14; yields Eq. 16.

$$
\begin{equation*}
\psi(\vec{r}, t)=\psi_{0} \exp \left(i\left(\frac{\vec{p} \cdot \vec{r}}{\hbar}-\omega t\right)\right) \tag{16}
\end{equation*}
$$

Substitute RHS of Eq. 16 into Eq. 9; yields Eq. 17.

$$
\begin{equation*}
\psi(\vec{r}, t)=\psi_{0} \exp \left(i\left(\frac{\vec{p} \cdot \vec{r}}{\hbar}-\frac{E t}{\hbar}\right)\right) \tag{17}
\end{equation*}
$$

Simplify Eq. 17; yields Eq. 18.

$$
\begin{equation*}
\psi(\vec{r}, t)=\psi_{0} \exp \left(\frac{i}{\hbar}(\vec{p} \cdot \vec{r}-E t)\right) \tag{18}
\end{equation*}
$$

Eq. 19 is an initial equation.

$$
\begin{equation*}
p=m v \tag{19}
\end{equation*}
$$

Eq. 20 is an initial equation.

$$
\begin{equation*}
E=\frac{1}{2} m v^{2} \tag{20}
\end{equation*}
$$

Raise both sides of Eq. 19 to 2; yields Eq. 21.

$$
\begin{equation*}
p^{2}=m^{2} v^{2} \tag{21}
\end{equation*}
$$

Multiply RHS of Eq. 20 by 1, which in this case is $m / m$; yields Eq. 22

$$
\begin{equation*}
E=\frac{1}{2 m} m^{2} v^{2} \tag{22}
\end{equation*}
$$

Substitute RHS of Eq. 21 into Eq. 22; yields Eq. 23.

$$
\begin{equation*}
E=\frac{p^{2}}{2 m} \tag{23}
\end{equation*}
$$

Partially differentiate Eq. 18 with respect to $t$; yields Eq. 24.

$$
\begin{equation*}
\frac{\partial}{\partial t} \psi(\vec{r}, t)=\psi_{0} \frac{\partial}{\partial t} \exp \left(i\left(\frac{\vec{p} \cdot \vec{r}}{\hbar}-\frac{E t}{\hbar}\right)\right) \tag{24}
\end{equation*}
$$

Substitute RHS of Eq. 24 into Eq. 18; yields Eq. 25.

$$
\begin{equation*}
\frac{\partial}{\partial t} \psi(\vec{r}, t)=\frac{-i}{\hbar} E \psi(\vec{r}, t) \tag{25}
\end{equation*}
$$

Substitute RHS of Eq. 23 into Eq. 25; yields Eq. 26.

$$
\begin{equation*}
\frac{\partial}{\partial t} \psi(\vec{r}, t)=\frac{-i}{\hbar} \frac{p^{2}}{2 m} \psi(\vec{r}, t) \tag{26}
\end{equation*}
$$

Multiply both sides of Eq. 26 by $i \hbar$; yields Eq. 27.

$$
\begin{equation*}
i \hbar \frac{\partial}{\partial t} \psi(\vec{r}, t)=\frac{p^{2}}{2 m} \psi(\vec{r}, t) \tag{27}
\end{equation*}
$$

Apply gradient to both sides of Eq. 18; yields Eq. 28.

$$
\begin{equation*}
\psi(\vec{r}, t)=\psi_{0} \exp \left(\frac{i}{\hbar}(\vec{p} \cdot \vec{r}-E t)\right) \tag{28}
\end{equation*}
$$

Simplify Eq. 28; yields Eq. 29.

$$
\begin{equation*}
\vec{\nabla} \psi(\vec{r}, t)=\frac{i}{\hbar} \vec{p} \psi_{0} \exp \left(\frac{i}{\hbar}(\vec{p} \cdot \vec{r}-E t)\right) \tag{29}
\end{equation*}
$$

Substitute RHS of Eq. 29 into Eq. 18; yields Eq. 30.

$$
\begin{equation*}
\vec{\nabla} \psi(\vec{r}, t)=\frac{i}{\hbar} \vec{p} \psi(\vec{r}, t) \tag{30}
\end{equation*}
$$

Apply divergence to both sides of Eq. 30; yields Eq. 31.

$$
\begin{equation*}
\vec{\nabla} \cdot(\vec{\nabla} \psi(\vec{r}, t))=\frac{i}{\hbar} \vec{\nabla} \cdot(\vec{p} \psi(\vec{r}, t)) \tag{31}
\end{equation*}
$$

Simplify Eq. 31; yields Eq. 32.

$$
\begin{equation*}
\nabla^{2} \psi(\vec{r}, t)=\frac{i}{\hbar} \vec{p} \cdot(\vec{\nabla} \psi(\vec{r}, t)) \tag{32}
\end{equation*}
$$

Substitute RHS of Eq. 30 into Eq. 32; yields Eq. 33.

$$
\begin{equation*}
\nabla^{2} \psi(\vec{r}, t)=\frac{i}{\hbar} \vec{p} \cdot\left(\frac{i}{\hbar} \vec{p} \psi(\vec{r}, t)\right) \tag{33}
\end{equation*}
$$

Simplify Eq. 33; yields Eq. 34.

$$
\begin{equation*}
\nabla^{2} \psi(\vec{r}, t)=\frac{-p^{2}}{\hbar} \psi(\vec{r}, t) \tag{34}
\end{equation*}
$$

Multiply both sides of Eq. 34 by $\frac{-\hbar^{2}}{2 m}$; yields Eq. 35.

$$
\begin{equation*}
\frac{-\hbar^{2}}{2 m} \nabla^{2} \psi(\vec{r}, t)=\frac{p^{2}}{2 m} \psi(\vec{r}, t) \tag{35}
\end{equation*}
$$

LHS of Eq. 27 is equal to LHS of Eq. 35; yields Eq. 36.

$$
\begin{equation*}
\frac{-\hbar^{2}}{2 m} \nabla^{2} \psi(\vec{r}, t)=i \hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) \tag{36}
\end{equation*}
$$

Eq. 37 is an initial equation.

$$
\begin{equation*}
\frac{-\hbar^{2}}{2 m} \nabla^{2}=\mathcal{H} \tag{37}
\end{equation*}
$$

Substitute LHS of Eq. 37 into Eq. 36; yields Eq. 38.

$$
\begin{equation*}
\mathcal{H} \psi(\vec{r}, t)=i \hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) \tag{38}
\end{equation*}
$$

Eq. 38 is one of the final equations.

## References

