derivation of Schrodinger Equation

none

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Abstract

Generated by the Physics Derivation Graph.

Eq. 1 is an initial equation.

$$k = \frac{2\pi}{\lambda} \tag{1}$$

Eq. 2 is an initial equation.

$$\omega = 2\pi f \tag{2}$$

Eq. 3 is an initial equation.

$$\hbar = h/(2\pi) \tag{3}$$

Eq. 4 is an initial equation.

$$p = h/\lambda \tag{4}$$

Eq. 5 is an initial equation.

$$E = hf \tag{5}$$

Divide both sides of Eq. 2 by 2π; yields Eq. 6.

$$\frac{\omega}{2\pi} = f \tag{6}$$

Substitute RHS of Eq. 6 into Eq. 5; yields Eq. 7.

$$E = \frac{h\omega}{2\pi} \tag{7}$$

Substitute RHS of Eq. 3 into Eq. 7; yields Eq. 8.

$$E = \omega\hbar \tag{8}$$

Divide both sides of Eq. 8 by $\hbar$; yields Eq. 9.

$$\frac{E}{\hbar} = \omega \tag{9}$$

Divide both sides of Eq. 1 by 2π; yields Eq. 10.

$$\frac{k}{2\pi} = \lambda \tag{10}$$

Substitute RHS of Eq. 4 into Eq. 10; yields Eq. 11.

$$p = \frac{hk}{2\pi} \tag{11}$$

Substitute RHS of Eq. 11 into Eq. 3; yields Eq. 12.

$$p = \hbar k \tag{12}$$

Divide both sides of Eq. 12 by $\hbar$; yields Eq. 13.

$$\frac{p}{\hbar} = k \tag{13}$$

Replace scalar variables in Eq. 13 with equivalent vector variables; yields Eq. 14.

$$\frac{\vec{p}}{\hbar} = \vec{k} \tag{14}$$

Eq. 15 is an initial equation.

$$\psi(\vec{r}, t) = \psi_0 \exp\left(i\left(\vec{k} \cdot \vec{r} - \omega t\right)\right) \tag{15}$$

Substitute RHS of Eq. 15 into Eq. 14; yields Eq. 16.

$$\psi(\vec{r}, t) = \psi_0 \exp\left(i\left(\frac{\vec{p} \cdot \vec{r}}{\hbar} - \omega t\right)\right) \tag{16}$$

Substitute RHS of Eq. 16 into Eq. 9; yields Eq. 17.

$$\psi(\vec{r}, t) = \psi_0 \exp\left(i\left(\frac{\vec{p} \cdot \vec{r}}{\hbar} - \frac{Et}{\hbar}\right)\right) \tag{17}$$

Simplify Eq. 17; yields Eq. 18.

$$\psi(\vec{r}, t) = \psi_0 \exp\left(\frac{i}{\hbar}\left(\vec{p} \cdot \vec{r} - Et\right)\right) \tag{18}$$

Eq. 19 is an initial equation.

$$p = mv \tag{19}$$

Eq. 20 is an initial equation.

$$E = \frac{1}{2}mv^2 \tag{20}$$

Raise both sides of Eq. 19 to 2; yields Eq. 21.

$$p^2 = m^2 v^2 \tag{21}$$

Multiply RHS of Eq. 20 by 1, which in this case is m/m; yields Eq. 22

$$E = \frac{1}{2m}m^2 v^2 \tag{22}$$

Substitute RHS of Eq. 21 into Eq. 22; yields Eq. 23.

$$E = \frac{p^2}{2m} \tag{23}$$

Partially differentiate Eq. 18 with respect to t; yields Eq. 24.

$$\frac{\partial}{\partial t}\psi(\vec{r},t) = \psi_0 \frac{\partial}{\partial t} \exp\left(i\left(\frac{\vec{p}\cdot\vec{r}}{\hbar} - \frac{Et}{\hbar}\right)\right) \tag{24}$$

Substitute RHS of Eq. 24 into Eq. 18; yields Eq. 25.

$$\frac{\partial}{\partial t}\psi(\vec{r},t) = \frac{-i}{\hbar} E\psi(\vec{r},t) \tag{25}$$

Substitute RHS of Eq. 23 into Eq. 25; yields Eq. 26.

$$\frac{\partial}{\partial t}\psi(\vec{r},t) = \frac{-i}{\hbar}\frac{p^2}{2m}\psi(\vec{r},t) \tag{26}$$

Multiply both sides of Eq. 26 by $i\hbar$; yields Eq. 27.

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \frac{p^2}{2m}\psi(\vec{r},t) \tag{27}$$

Apply gradient to both sides of Eq. 18; yields Eq. 28.

$$\psi(\vec{r},t) = \psi_0 \exp\left(\frac{i}{\hbar}\left(\vec{p}\cdot\vec{r} - Et\right)\right) \tag{28}$$

Simplify Eq. 28; yields Eq. 29.

$$\vec{\nabla}\psi(\vec{r},t) = \frac{i}{\hbar}\vec{p}\psi_0 \exp\left(\frac{i}{\hbar}\left(\vec{p}\cdot\vec{r} - Et\right)\right) \tag{29}$$

Substitute RHS of Eq. 29 into Eq. 18; yields Eq. 30.

$$\vec{\nabla}\psi(\vec{r},t) = \frac{i}{\hbar}\vec{p}\psi(\vec{r},t) \tag{30}$$

Apply divergence to both sides of Eq. 30; yields Eq. 31.

$$\vec{\nabla}\cdot\left(\vec{\nabla}\psi(\vec{r},t)\right) = \frac{i}{\hbar}\vec{\nabla}\cdot\left(\vec{p}\psi(\vec{r},t)\right) \tag{31}$$

Simplify Eq. 31; yields Eq. 32.

$$\nabla^2\psi\left(\vec{r},t\right) = \frac{i}{\hbar}\vec{p}\cdot\left(\vec{\nabla}\psi(\vec{r},t)\right) \tag{32}$$

Substitute RHS of Eq. 30 into Eq. 32; yields Eq. 33.

$$\nabla^2\psi\left(\vec{r},t\right) = \frac{i}{\hbar}\vec{p}\cdot\left(\frac{i}{\hbar}\vec{p}\psi(\vec{r},t)\right) \tag{33}$$

Simplify Eq. 33; yields Eq. 34.

$$\nabla^2\psi\left(\vec{r},t\right)=\frac{-p^2}{\hbar}\psi(\vec{r},t) \tag{34}$$

Multiply both sides of Eq. 34 by $\frac{-\hbar^2}{2m}$; yields Eq. 35.

$$\frac{-\hbar^2}{2m}\nabla^2\psi\left(\vec{r},t\right)=\frac{p^2}{2m}\psi(\vec{r},t) \tag{35}$$

LHS of Eq. 27 is equal to LHS of Eq. 35; yields Eq. 36.

$$\frac{-\hbar^2}{2m}\nabla^2\psi\left(\vec{r},t\right)=i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) \tag{36}$$

Eq. 37 is an initial equation.

$$\frac{-\hbar^2}{2m}\nabla^2=\mathcal{H} \tag{37}$$

Substitute LHS of Eq. 37 into Eq. 36; yields Eq. 38.

$$\mathcal{H}\psi\left(\vec{r},t\right)=i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) \tag{38}$$

Eq. 38 is one of the final equations.

References