electric field wave equation: from time dependent to time independent

none

March 16, 2024

Abstract

Generated by the Physics Derivation Graph.

Eq. 1 is an initial equation.

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{1}$$

Eq. 2 is an initial equation.

$$\vec{E} = E(\vec{r}, t) \tag{2}$$

Judicious choice as a guessed solution to Eq. 1 is Eq. 3,

$$E(\vec{r}, t) = E(\vec{r}) \exp(i\omega t) \tag{3}$$

Substitute LHS of Eq. 2 into Eq. 1; yields Eq. 4.

$$\nabla^2 E(\vec{r}, t) = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E(\vec{r}, t) \tag{4}$$

Substitute LHS of Eq. 3 into Eq. 4; yields Eq. 5.

$$\nabla^2 E(\vec{r}) \exp(i\omega t) = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} E(\vec{r}) \exp(i\omega t) \tag{5}$$

Differentiate Eq. 5 with respect to t; yields Eq. 6.

$$\nabla^2 E(\vec{r}) \exp(i\omega t) = i\omega \mu_0 \epsilon_0 \frac{\partial}{\partial t} E(\vec{r}) \exp(i\omega t) \tag{6}$$

Differentiate Eq. 6 with respect to t; yields Eq. 7.

$$\nabla^2 E(\vec{r}) \exp(i\omega t) = -\omega^2 \mu_0 \epsilon_0 E(\vec{r}) \exp(i\omega t) \tag{7}$$

Eq. 8 is an initial equation.

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \tag{8}$$

Substitute LHS of Eq. 7 into Eq. 8; yields Eq. 9.

$$\nabla^2 E(\vec{r}) \exp(i\omega t) = -\frac{\omega^2}{c^2} E(\vec{r}) \exp(i\omega t) \tag{9}$$

Simplify Eq. 9; yields Eq. 10.

$$\nabla^2 E(\vec{r}) = -\frac{\omega^2}{c^2} E(\vec{r}) \tag{10}$$

Eq. 10 is one of the final equations.

References