# particle in a 1D box 

none

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#### Abstract

Generated by the Physics Derivation Graph.


Eq. 1 is an initial equation.

$$
\begin{equation*}
\frac{d^{2}}{d x^{2}} \psi(x)=-k^{2} \psi(x) \tag{1}
\end{equation*}
$$

Judicious choice as a guessed solution to Eq. 1 is Eq. 2,

$$
\begin{equation*}
\psi(x)=a \sin (k x)+b \cos (k x) \tag{2}
\end{equation*}
$$

A boundary condition for Eq. 1 is Eq. 3

$$
\begin{equation*}
\psi(x=0)=0 \tag{3}
\end{equation*}
$$

A boundary condition for Eq. 1 is Eq. 4

$$
\begin{equation*}
\psi(x=W)=0 \tag{4}
\end{equation*}
$$

LHS of Eq. 3 is equal to LHS of Eq. 2; yields Eq. 5.

$$
\begin{equation*}
0=a \sin (0)+b \cos (0) \tag{5}
\end{equation*}
$$

Simplify Eq. 5; yields Eq. 6.

$$
\begin{equation*}
0=b \tag{6}
\end{equation*}
$$

Substitute RHS of Eq. 6 into Eq. 2; yields Eq. 7.

$$
\begin{equation*}
\psi(x)=a \sin (k x) \tag{7}
\end{equation*}
$$

LHS of Eq. 4 is equal to LHS of Eq. 7; yields Eq. 8.

$$
\begin{equation*}
0=a \sin (k W) \tag{8}
\end{equation*}
$$

Eq. 9 is an identity.

$$
\begin{equation*}
0=a \sin (n \pi) \tag{9}
\end{equation*}
$$

Eq. 8 is valid when Eq. 9 occurs; yields Eq. 10 .

$$
\begin{equation*}
k W=n \pi \tag{10}
\end{equation*}
$$

Divide both sides of Eq. 10 by $W$; yields Eq. 11.

$$
\begin{equation*}
k=\frac{n \pi}{W} \tag{11}
\end{equation*}
$$

Substitute RHS of Eq. 11 into Eq. 7; yields Eq. 12.

$$
\begin{equation*}
\psi(x)=a \sin \left(\frac{n \pi}{W} x\right) \tag{12}
\end{equation*}
$$

Normalization condition is Eq. 13.

$$
\begin{equation*}
\int|\psi(x)|^{2} d x=1 \tag{13}
\end{equation*}
$$

Conjugate $\psi$ in Eq. 12; yields Eq. 14.

$$
\begin{equation*}
\psi(x)^{*}=a \sin \left(\frac{n \pi}{W} x\right) \tag{14}
\end{equation*}
$$

Swap LHS of Eq. 13 with RHS; yields Eq. 15.

$$
\begin{equation*}
1=\int|\psi(x)|^{2} d x \tag{15}
\end{equation*}
$$

Expand $\psi(x)$ in Eq. 15 with conjugate; yields Eq. 16.

$$
\begin{equation*}
1=\int \psi(x) \psi(x)^{*} d x \tag{16}
\end{equation*}
$$

Substitute LHS of Eq. 12 into Eq. 16; yields Eq. 17.

$$
\begin{equation*}
1=\int_{0}^{W} a \sin \left(\frac{n \pi}{W} x\right) \psi(x)^{*} d x \tag{17}
\end{equation*}
$$

Substitute LHS of Eq. 14 into Eq. 17; yields Eq. 18.

$$
\begin{equation*}
1=\int_{0}^{W} a^{2}\left(\sin \left(\frac{n \pi}{W} x\right)\right)^{2} d x \tag{18}
\end{equation*}
$$

Eq. 19 is an identity.

$$
\begin{equation*}
(\sin (x))^{2}=\frac{1-\cos (2 x)}{2} \tag{19}
\end{equation*}
$$

Change variable $\frac{n \pi}{W} x$ to $x$ in Eq. 19; yields Eq. 20.

$$
\begin{equation*}
\left(\sin \left(\frac{n \pi}{W} x\right)\right)^{2}=\frac{1-\cos \left(2 \frac{n \pi}{W} x\right)}{2} \tag{20}
\end{equation*}
$$

Substitute RHS of Eq. 20 into Eq. 18; yields Eq. 21.

$$
\begin{equation*}
1=\int_{0}^{W} a^{2} \frac{1-\cos \left(2 \frac{n \pi}{W} x\right)}{2} d x \tag{21}
\end{equation*}
$$

Divide both sides of Eq. 21 by $a^{2}$; yields Eq. 22.

$$
\begin{equation*}
\frac{1}{a^{2}}=\int_{0}^{W} \frac{1-\cos \left(2 \frac{n \pi}{W} x\right)}{2} d x \tag{22}
\end{equation*}
$$

Expand integrand of Eq. 22; yields Eq. 23.

$$
\begin{equation*}
\frac{1}{a^{2}}=\int_{0}^{W} \frac{1}{2} d x-\frac{1}{2} \int_{0}^{W} \cos \left(2 \frac{n \pi}{W} x\right) d x \tag{23}
\end{equation*}
$$

Eq. 24 is an identity.

$$
\begin{equation*}
\int \cos (a x) d x=\frac{1}{a} \sin (a x) \tag{24}
\end{equation*}
$$

Change variable $\frac{2 n \pi}{W}$ to $a$ in Eq. 24; yields Eq. 25.

$$
\begin{equation*}
\int \cos \left(\frac{2 n \pi}{W} x\right) d x=\frac{W}{2 n \pi} \sin \left(\frac{2 n \pi}{W} x\right) \tag{25}
\end{equation*}
$$

Eq. 26 is an identity.

$$
\begin{equation*}
\int a d x=a x \tag{26}
\end{equation*}
$$

Change variable 1/2 to $a$ in Eq. 26; yields Eq. 27.

$$
\begin{equation*}
\int \frac{1}{2} d x=\frac{1}{2} x \tag{27}
\end{equation*}
$$

Substitute LHS of Eq. 27 into Eq. 23; yields Eq. 28.

$$
\begin{equation*}
\frac{1}{a^{2}}=\frac{1}{2} W-\frac{1}{2} \int_{0}^{W} \cos \left(2 \frac{n \pi}{W} x\right) d x \tag{28}
\end{equation*}
$$

Substitute RHS of Eq. 25 into Eq. 28; yields Eq. 29.

$$
\begin{equation*}
\frac{1}{a^{2}}=\frac{1}{2} W-\left.\frac{1}{2} \frac{W}{2 n \pi} \sin \left(\frac{2 n \pi}{W} x\right)\right|_{0} ^{W} \tag{29}
\end{equation*}
$$

Simplify Eq. 29; yields Eq. 30.

$$
\begin{equation*}
\frac{1}{a^{2}}=\frac{W}{2} \tag{30}
\end{equation*}
$$

Multiply both sides of Eq. 30 by $a^{2} \frac{2}{W}$; yields Eq. 31.

$$
\begin{equation*}
\frac{2}{W}=a^{2} \tag{31}
\end{equation*}
$$

Take the square root of both sides of Eq. 31; yields Eq. 32 and Eq. 33.

$$
\begin{equation*}
\sqrt{\frac{2}{W}}=a \tag{32}
\end{equation*}
$$

$$
\begin{equation*}
-\sqrt{\frac{2}{W}}=a \tag{33}
\end{equation*}
$$

Substitute LHS of Eq. 32 into Eq. 12; yields Eq. 34.

$$
\begin{equation*}
\psi(x)=-\sqrt{\frac{2}{W}} \sin \left(\frac{n \pi}{W} x\right) \tag{34}
\end{equation*}
$$

Substitute LHS of Eq. 33 into Eq. 12; yields Eq. 35.

$$
\begin{equation*}
\psi(x)=\sqrt{\frac{2}{W}} \sin \left(\frac{n \pi}{W} x\right) \tag{35}
\end{equation*}
$$

Eq. 35 is one of the final equations. Substitute RHS of Eq. 1 into Eq. 2; yields Eq. 36.

$$
\begin{equation*}
\frac{d^{2}}{d x^{2}}(a \sin (k x)+b \cos (k x))=-k^{2}(a \sin (k x)+b \cos (k x)) \tag{36}
\end{equation*}
$$

Simplify Eq. 36; yields Eq. 37.

$$
\begin{equation*}
a \frac{d^{2}}{d x^{2}} \sin (k x)+b \frac{d^{2}}{d x^{2}} \cos (k x)=-a k^{2} \sin (k x)+-b k^{2} \cos (k x) \tag{37}
\end{equation*}
$$

Simplify Eq. 37; yields Eq. 38.

$$
\begin{equation*}
-a k^{2} \sin (k x)+-b k^{2} \cos (k x)=-a k^{2} \sin (k x)+-b k^{2} \cos (k x) \tag{38}
\end{equation*}
$$

Thus we see that LHS of Eq. 38 is equal to RHS.

## References

