equations of motion in 2D (calculus)

none

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Abstract

Generated by the Physics Derivation Graph.

Eq. **??** is an initial equation.

$$\vec{a} = \frac{d\vec{v}}{dt} \tag{1}$$

Substitute LHS of Eq. **??** into Eq. **??**; yields Eq. **??**.

$$a_x\hat{x} + a_y\hat{y} = \frac{d\vec{v}}{dt} \tag{2}$$

Substitute LHS of Eq. **??** into Eq. **??**; yields Eq. **??**.

$$a_x\hat{x} + a_y\hat{y} = \frac{d}{dt}\left(v_x\hat{x} + v_y\hat{y}\right) \tag{3}$$

Separate two vector components in Eq. **??**; yields Eq. **??** and Eq. **??**

$$a_x = \frac{d}{dt}v_x \tag{4}$$

$$a_y = \frac{d}{dt}v_y \tag{5}$$

Eq. **??** is an assumption. define the orientation of the coordinate system with respect to the gravitational acceleration such that x axis is perpendicular to gravity

$$a_x = 0 \tag{6}$$

Eq. **??** is an assumption. define the orientation of the coordinate system with respect to the gravitational acceleration such that y axis is parallel to gravity

$$a_y = -g \tag{7}$$

Assume 2 dimensions; decompose vector to be Eq. **??**.

$$\vec{a} = a_x\hat{x} + a_y\hat{y} \tag{8}$$

Assume 2 dimensions; decompose vector to be Eq. **??**.

$$\vec{v} = v_x \hat{x} + v_y \hat{y} \tag{9}$$

Substitute LHS of Eq. **??** into Eq. **??**; yields Eq. **??**.

$$0 = \frac{d}{dt} v_x \tag{10}$$

Substitute LHS of Eq. **??** into Eq. **??**; yields Eq. **??**.

$$-g = \frac{d}{dt} v_y \tag{11}$$

Multiply both sides of Eq. **??** by dt; yields Eq. **??**.

$$-g dt = dv_y \tag{12}$$

Indefinite integral of both sides of Eq. **??**; yields Eq. **??**.

$$-g \int dt = \int dv_y \tag{13}$$

Simplify Eq. **??**; yields Eq. **??**.

$$-gt = v_y - v_{0,y} \tag{14}$$

Add $v_{0,y}$ to both sides of Eq. **??**; yields Eq. **??**.

$$-gt + v_{0,y} = v_y \tag{15}$$

Eq. **??** is an initial equation.

$$v_y = \frac{dy}{dt} \tag{16}$$

Substitute LHS of Eq. **??** into Eq. **??**; yields Eq. **??**.

$$-gt + v_{0,y} = \frac{dy}{dt} \tag{17}$$

Multiply both sides of Eq. **??** by dt; yields Eq. **??**.

$$-gtdt + v_{0,y} dt = dy \tag{18}$$

Indefinite integral of both sides of Eq. **??**; yields Eq. **??**.

$$-g \int tdt + v_{0,y} \int dt = \int dy \tag{19}$$

Simplify Eq. **??**; yields Eq. **??**.

$$-\frac{1}{2} g t^2 + v_{0,y} t = y - y_0 \tag{20}$$

Add y_0 to both sides of Eq. **??**; yields Eq. **??**.

$$-\frac{1}{2}gt^2 + v_{0,y}t + y_0 = y \tag{21}$$

Multiply both sides of Eq. **??** by dt; yields Eq. **??**.

$$0dt = dv_x \tag{22}$$

Indefinite integral of both sides of Eq. **??**; yields Eq. **??**.

$$\int 0dt = \int dv_x \tag{23}$$

Simplify Eq. **??**; yields Eq. **??**.

$$0 = v_x - v_{0,x} \tag{24}$$

Add $v_{0,x}$ to both sides of Eq. **??**; yields Eq. **??**.

$$v_{0,x} = v_x \tag{25}$$

Eq. **??** is an initial equation.

$$v_x = \frac{dx}{dt} \tag{26}$$

Substitute LHS of Eq. **??** into Eq. **??**; yields Eq. **??**.

$$v_{0,x} = \frac{dx}{dt} \tag{27}$$

Multiply both sides of Eq. **??** by dt; yields Eq. **??**.

$$v_{0,x}dt = dx \tag{28}$$

Indefinite integral of both sides of Eq. **??**; yields Eq. **??**.

$$v_{0,x}\int dt = \int dx \tag{29}$$

Simplify Eq. **??**; yields Eq. **??**.

$$v_{0,x}t = x - x_0 \tag{30}$$

Add x_0 to both sides of Eq. **??**; yields Eq. **??**.

$$v_{0,x}t + x_0 = x \tag{31}$$

Swap LHS of Eq. **??** with RHS; yields Eq. **??**.

$$x = v_{0,x}t + x_0 \tag{32}$$

Assume 2 dimensions; decompose vector to be Eq. **??**.

$$\vec{v}_0 = v_{0,x}\hat{x} + v_{0,y}\hat{y} \tag{33}$$

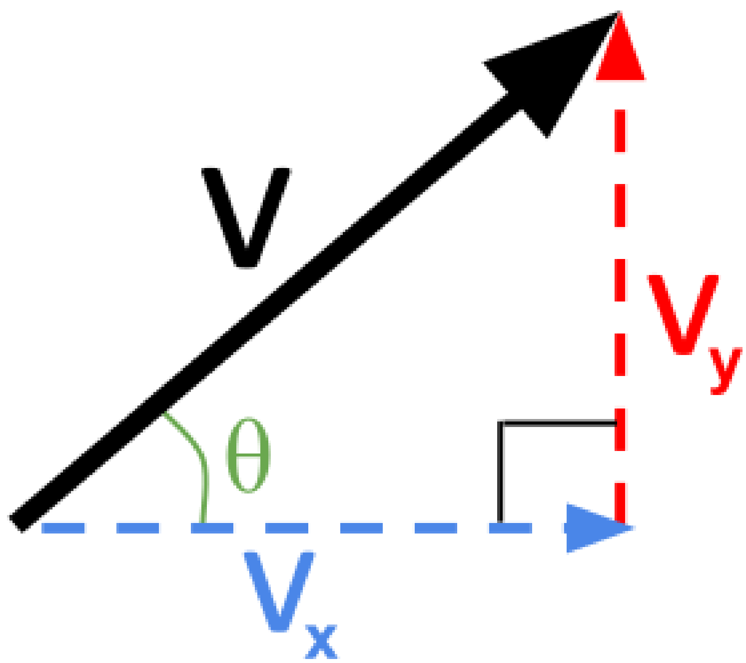

Figure 1: vector v components

Separate vector in Eq. ?? into components related by angle θ; yields Eq. ?? and Eq. ??.

$$\cos(\theta) = \frac{v_{0,x}}{v_0} \tag{34}$$

$$\sin(\theta) = \frac{v_{0,y}}{v_0} \tag{35}$$

Multiply both sides of Eq. ?? by v_0; yields Eq. ??.

$$v_0 \cos(\theta) = v_{0,x} \tag{36}$$

Substitute LHS of Eq. ?? into Eq. ??; yields Eq. ??.

$$x = v_0 t \cos(\theta) + x_0 \tag{37}$$

Eq. ?? is one of the final equations. Multiply both sides of Eq. ?? by v_0; yields Eq. ??.

$$v_0 \sin(\theta) = v_{0,y} \tag{38}$$

Swap LHS of Eq. ?? with RHS; yields Eq. ??.

$$y = -\frac{1}{2} g t^2 + v_{0,y} t + y_0 \tag{39}$$

Substitute LHS of Eq. ?? into Eq. ??; yields Eq. ??.

$$y = -\frac{1}{2} g t^2 + v_0 t \sin(\theta) + y_0 \tag{40}$$

Eq. ?? is one of the final equations.