# angle of maximum distance for projectile motion 

none

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#### Abstract

Generated by the Physics Derivation Graph. Eq. 1 is an initial equation. $$
\begin{equation*} y=-\frac{1}{2} g t^{2}+v_{0} t \sin (\theta)+y_{0} \tag{1} \end{equation*}
$$


Change variable $y$ to $y_{f}$ and $t$ to $t_{f}$ in Eq. 1; yields Eq. 2.

$$
\begin{equation*}
y_{f}=-\frac{1}{2} g t_{f}^{2}+v_{0} t_{f} \sin (\theta)+y_{0} \tag{2}
\end{equation*}
$$

Boundary condition: Eq. 3 when Eq. ??. $y\left(t \_f\right)=y \_f=0$

$$
\begin{equation*}
y_{f}=0 \tag{3}
\end{equation*}
$$

LHS of Eq. 2 is equal to LHS of Eq. 3; yields Eq. 4.

$$
\begin{equation*}
0=-\frac{1}{2} g t_{f}^{2}+v_{0} t_{f} \sin (\theta)+y_{0} \tag{4}
\end{equation*}
$$

Eq. 5 is an assumption.

$$
\begin{equation*}
y_{0}=0 \tag{5}
\end{equation*}
$$

Substitute LHS of Eq. 4 into Eq. 5; yields Eq. 6.

$$
\begin{equation*}
0=-\frac{1}{2} g t_{f}^{2}+v_{0} t_{f} \sin (\theta) \tag{6}
\end{equation*}
$$

Divide both sides of Eq. 6 by $t_{f}$; yields Eq. 7 .

$$
\begin{equation*}
0=-\frac{1}{2} g t_{f}+v_{0} \sin (\theta) \tag{7}
\end{equation*}
$$

Add $\frac{1}{2} g t_{f}$ to both sides of Eq. 7; yields Eq. 8.

$$
\begin{equation*}
\frac{1}{2} g t_{f}=v_{0} \sin (\theta) \tag{8}
\end{equation*}
$$

Multiply both sides of Eq. 8 by $2 / g$; yields Eq. 9 .

$$
\begin{equation*}
t_{f}=\frac{2 v_{0} \sin (\theta)}{g} \tag{9}
\end{equation*}
$$

Eq. 10 is an initial equation.

$$
\begin{equation*}
x=v_{0} t \cos (\theta)+x_{0} \tag{10}
\end{equation*}
$$

Change variable $x$ to $x_{f}$ and $t$ to $t_{f}$ in Eq. 10; yields Eq. 11.

$$
\begin{equation*}
x_{f}=v_{0} t_{f} \cos (\theta)+x_{0} \tag{11}
\end{equation*}
$$

Boundary condition: Eq. 12 when Eq. ??.

$$
\begin{equation*}
x_{f}=x_{0}+d \tag{12}
\end{equation*}
$$

Substitute LHS of Eq. 12 into Eq. 11; yields Eq. 13.

$$
\begin{equation*}
x_{0}+d=v_{0} t_{f} \cos (\theta)+x_{0} \tag{13}
\end{equation*}
$$

Subtract $x_{0}$ from both sides of Eq. 13 ; yields Eq. 14.

$$
\begin{equation*}
d=v_{0} t_{f} \cos (\theta) \tag{14}
\end{equation*}
$$

Substitute LHS of Eq. 9 into Eq. 14; yields Eq. 15.

$$
\begin{equation*}
d=v_{0} \frac{2 v_{0} \sin (\theta)}{g} \cos (\theta) \tag{15}
\end{equation*}
$$

Eq. 16 is an initial equation.

$$
\begin{equation*}
\sin (2 x)=2 \sin (x) \cos (x) \tag{16}
\end{equation*}
$$

Change variable $\theta$ to $x$ in Eq. 16; yields Eq. 17.

$$
\begin{equation*}
\sin (2 \theta)=2 \sin (\theta) \cos (\theta) \tag{17}
\end{equation*}
$$

Substitute LHS of Eq. 17 into Eq. 15; yields Eq. 18.

$$
\begin{equation*}
d=\frac{v_{0}^{2}}{g} \sin (2 \theta) \tag{18}
\end{equation*}
$$

The maximum of Eq. 18 with respect to $\theta$ is Eq. 19

$$
\begin{equation*}
\theta=\frac{\pi}{4} \tag{19}
\end{equation*}
$$

Substitute LHS of Eq. 19 into Eq. 18; yields Eq. 20.

$$
\begin{equation*}
d=\frac{v_{0}^{2}}{g} \sin \left(2 \frac{\pi}{4}\right) \tag{20}
\end{equation*}
$$

Simplify Eq. 20; yields Eq. 21.

$$
\begin{equation*}
d=\frac{v_{0}^{2}}{g} \tag{21}
\end{equation*}
$$

Eq. 21 is one of the final equations. Eq. 19 is one of the final equations.

## References

