angle of maximum distance for projectile motion

none

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Abstract

Generated by the Physics Derivation Graph.

Eq. 1 is an initial equation.

$$y = -\frac{1}{2}gt^2 + v_0 t \sin(\theta) + y_0 \tag{1}$$

Change variable y to y_f and t to t_f in Eq. 1; yields Eq. 2.

$$y_f = -\frac{1}{2}gt_f^2 + v_0 t_f \sin(\theta) + y_0 \tag{2}$$

Boundary condition: Eq. 3 when Eq. **??**. y(t_f) = y_f = 0

$$y_f = 0 \tag{3}$$

LHS of Eq. 2 is equal to LHS of Eq. 3; yields Eq. 4.

$$0 = -\frac{1}{2}gt_f^2 + v_0 t_f \sin(\theta) + y_0 \tag{4}$$

Eq. 5 is an assumption.

$$y_0 = 0 \tag{5}$$

Substitute LHS of Eq. 4 into Eq. 5; yields Eq. 6.

$$0 = -\frac{1}{2}gt_f^2 + v_0 t_f \sin(\theta) \tag{6}$$

Divide both sides of Eq. 6 by t_f; yields Eq. 7.

$$0 = -\frac{1}{2}gt_f + v_0 \sin(\theta) \tag{7}$$

Add $\frac{1}{2}gt_f$ to both sides of Eq. 7; yields Eq. 8.

$$\frac{1}{2}gt_f = v_0 \sin(\theta) \tag{8}$$

Multiply both sides of Eq. 8 by $2/g$; yields Eq. 9.

$$t_f = \frac{2v_0 \sin(\theta)}{g} \tag{9}$$

Eq. 10 is an initial equation.

$$x = v_0 t \cos(\theta) + x_0 \tag{10}$$

Change variable x to x_f and t to t_f in Eq. 10; yields Eq. 11.

$$x_f = v_0 t_f \cos(\theta) + x_0 \tag{11}$$

Boundary condition: Eq. 12 when Eq. **??**.

$$x_f = x_0 + d \tag{12}$$

Substitute LHS of Eq. 12 into Eq. 11; yields Eq. 13.

$$x_0 + d = v_0 t_f \cos(\theta) + x_0 \tag{13}$$

Subtract x_0 from both sides of Eq. 13; yields Eq. 14.

$$d = v_0 t_f \cos(\theta) \tag{14}$$

Substitute LHS of Eq. 9 into Eq. 14; yields Eq. 15.

$$d = v_0 \frac{2v_0 \sin(\theta)}{g} \cos(\theta) \tag{15}$$

Eq. 16 is an initial equation.

$$\sin(2x) = 2\sin(x)\cos(x) \tag{16}$$

Change variable θ to x in Eq. 16; yields Eq. 17.

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \tag{17}$$

Substitute LHS of Eq. 17 into Eq. 15; yields Eq. 18.

$$d = \frac{v_0^2}{g} \sin(2\theta) \tag{18}$$

The maximum of Eq. 18 with respect to θ is Eq. 19

$$\theta = \frac{\pi}{4} \tag{19}$$

Substitute LHS of Eq. 19 into Eq. 18; yields Eq. 20.

$$d = \frac{v_0^2}{g} \sin\left(2\frac{\pi}{4}\right) \tag{20}$$

Simplify Eq. 20; yields Eq. 21.

$$d = \frac{v_0^2}{g} \tag{21}$$

Eq. 21 is one of the final equations. Eq. 19 is one of the final equations.

References