

curl curl identity

April 1, 2026

Abstract

Generated by the [Physics Derivation Graph](#).

Eq. 1 is an identity.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \quad (1)$$

Replace curl in Eq. 1 with Levi-Cevita contravariant; yields Eq. 2.

$$\epsilon^{i,j,k} \hat{x}_i \nabla_j (\vec{\nabla} \times \vec{E})_k = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \quad (2)$$

Replace curl in Eq. 2 with Levi-Cevita contravariant; yields Eq. 3.

$$\epsilon^{i,j,k} \epsilon_{n,j,k} \hat{x}_i \nabla_j \nabla^m E^n = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \quad (3)$$

Eq. 4 is an identity.

$$\epsilon^{i,j,k} \epsilon_{n,j,k} = \delta^l_j \delta^m_k - \delta^l_k \delta^m_j \quad (4)$$

Substitute RHS of Eq. 3 into Eq. 4; yields Eq. 5.

$$(\delta^l_j \delta^m_k - \delta^l_k \delta^m_j) \hat{x}_i \nabla_j \nabla^m E^n = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \quad (5)$$

Simplify Eq. 5; yields Eq. 6.

$$(\delta^l_j \delta^m_k \hat{x}_i \nabla_j \nabla^m E^n) - (\delta^l_k \delta^m_j \hat{x}_i \nabla_j \nabla^m E^n) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \quad (6)$$

Simplify Eq. 6; yields Eq. 7.

$$\hat{x}_m \nabla_n \nabla^m E^n - \hat{x}_n \nabla_m \nabla^m E^n = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \quad (7)$$

Replace summation notation in Eq. 7 with vector notation; yields Eq. 8.

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E} - \nabla^2 \vec{E}) \quad (8)$$

Thus we see that LHS of Eq. 8 is equal to RHS.

References