escape velocity

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Abstract

Generated by the Physics Derivation Graph.

Eq. 1 is an initial equation.

$$F = G\frac{m_1 m_2}{x^2} \tag{1}$$

Substitute LHS of Eq. **??** into Eq. 1; yields Eq. 2.

$$dW = G\frac{m_1 m_2}{x^2} dx \tag{2}$$

Integrate Eq. refeq:2d5a719e01b4155d8138d87a649b065d983c98295787b71f5d6f711d941ced2e; yields Eq. refeq:b0c3dff2411522dbfe1bf7515dba8864f026f201bac3dd97e6a050e6aae600ee.

$$\int dW = G m_1 m_2 \int_{r_{\text{Earth}}}^{\infty} \frac{1}{x^2} dx \tag{3}$$

Evaluate definite integral Eq. 3; yields Eq. 4.

$$W = G m_1 m_2 \left(\left. \frac{1}{x} \right|_{r_{\text{Earth}}}^{\infty} \right) \tag{4}$$

Change variable m to m_2 and m_{Earth} to m_1 in Eq. 4; yields Eq. 5.

$$W = G m_{\text{Earth}} m \left(\left. \frac{1}{x^2} \right|_{r_{\text{Earth}}}^{\infty} \right) \tag{5}$$

Simplify Eq. 5; yields Eq. 6.

$$W = G m_{\text{Earth}} m \left(0 - \frac{-1}{r_{\text{Earth}}} \right) \tag{6}$$

Simplify Eq. 6; yields Eq. 7.

$$W = G\frac{m_{\text{Earth}} m}{r_{\text{Earth}}} \tag{7}$$

Substitute LHS of Eq. **??** and LHS of Eq. **??** into Eq. **??**; yields Eq. 8.

$$KE_2 + PE_2 = KE_1 + PE_1 \tag{8}$$

Eq. 9 is an assumption.

$$PE_2 = 0 \tag{9}$$

Eq. 10 is an assumption.

$$KE_2 = 0 \tag{10}$$

Substitute LHS of Eq. 8 and LHS of Eq. 10 into Eq. 9; yields Eq. 11.

$$0 = KE_1 + PE_1 \tag{11}$$

Change variable $PE_{\text{Earth surface}}$ to PE_1 and KE_{escape} to KE_1 in Eq. 11; yields Eq. 12.

$$0 = KE_{\text{escape}} + PE_{\text{Earth surface}} \tag{12}$$

Eq. 13 is an initial equation.

$$PE_{\text{Earth surface}} = -W \tag{13}$$

Substitute LHS of Eq. 13 into Eq. 7; yields Eq. 14.

$$PE_{\text{Earth surface}} = -G\frac{m_{\text{Earth}}m}{r_{\text{Earth}}} \tag{14}$$

Change variable v_{escape} to v and KE_{escape} to KE in Eq. **??**; yields Eq. 15.

$$KE_{\text{escape}} = \frac{1}{2}mv_{\text{escape}}^2 \tag{15}$$

Substitute LHS of Eq. 12 and LHS of Eq. 14 into Eq. 15; yields Eq. 16.

$$0 = -G\frac{m_{\text{Earth}}m}{r_{\text{Earth}}} + \frac{1}{2}mv_{\text{escape}}^2 \tag{16}$$

Add $G\frac{m_{\text{Earth}}m}{r_{\text{Earth}}}$ to both sides of Eq. 16; yields Eq. 17.

$$G\frac{m_{\text{Earth}}m}{r_{\text{Earth}}} = \frac{1}{2}mv_{\text{escape}}^2 \tag{17}$$

Simplify Eq. 17; yields Eq. 18.

$$G\frac{m_{\text{Earth}}}{r_{\text{Earth}}} = \frac{1}{2}v_{\text{escape}}^2 \tag{18}$$

Multiply both sides of Eq. 18 by 2; yields Eq. 19.

$$2G\frac{m_{\text{Earth}}}{r_{\text{Earth}}} = v_{\text{escape}}^2 \tag{19}$$

Swap LHS of Eq. 19 with RHS; yields Eq. 20.

$$v_{\text{escape}}^2 = 2G\frac{m_{\text{Earth}}}{r_{\text{Earth}}} \tag{20}$$

Take the square root of both sides of Eq. 20; yields Eq. 21 and Eq. 22.

$$v_{\text{escape}} = -\sqrt{2G\frac{m_{\text{Earth}}}{r_{\text{Earth}}}} \tag{21}$$

$$v_{\text{escape}} = \sqrt{2G\frac{m_{\text{Earth}}}{r_{\text{Earth}}}} \tag{22}$$

Eq. 22 is one of the final equations.

replaced Earth-specific variables Change variable r to r_{Earth} and m to m_{Earth} in Eq. 22; yields Eq. 23.

$$v_{\text{escape}} = \sqrt{2G\frac{m}{r}} \tag{23}$$

References